Washington Latin -- Honors Pre-Calculus

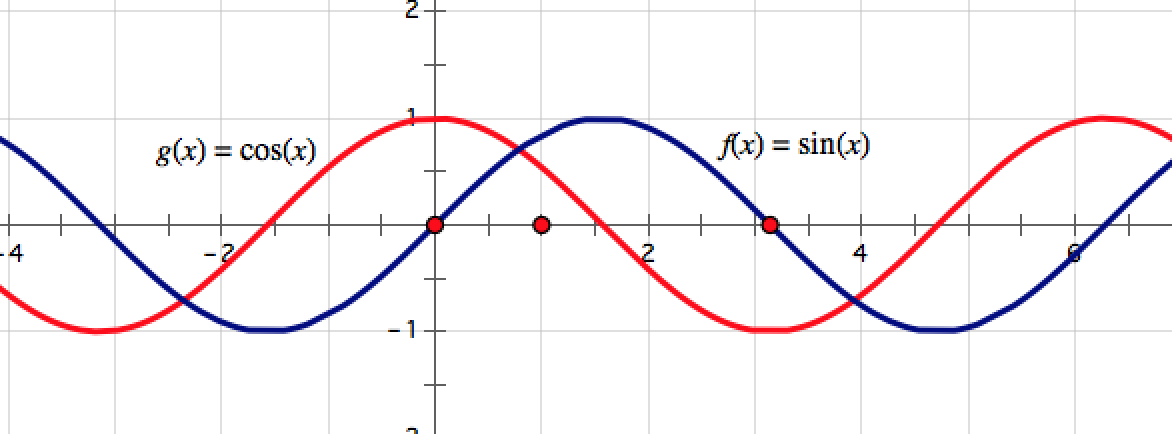
Tools for Graphing Sine and Cosine Curves

Here are the basics tools and steps for graphing sine and cosine curves. Remember two things.

1. Cosine curves are the same as sine curves. The basic cosine curve is simply a basic sine curve shifted π/2 to the left. Rather than starting at *(0, 0)* as the sine does, the cosine starts at *(0, 1).*

2. All sine curves are essentially the same. All you really are doing is changing the scale (amplitude and period), and moving it left or right, up or down on the coordinate plane.

Here are our two basic curves.



We are working on the general sine curve:

We look at each of the values, *a, b, h* and *k,* which determine everything about our sine curve.  *X* is our independent variable and *y* is our dependent variable. As we have seen, often *θ* is used to represent angles, which is what makes up the domain of sine curves, but we will stick with *x*.

1. **Amplitude**

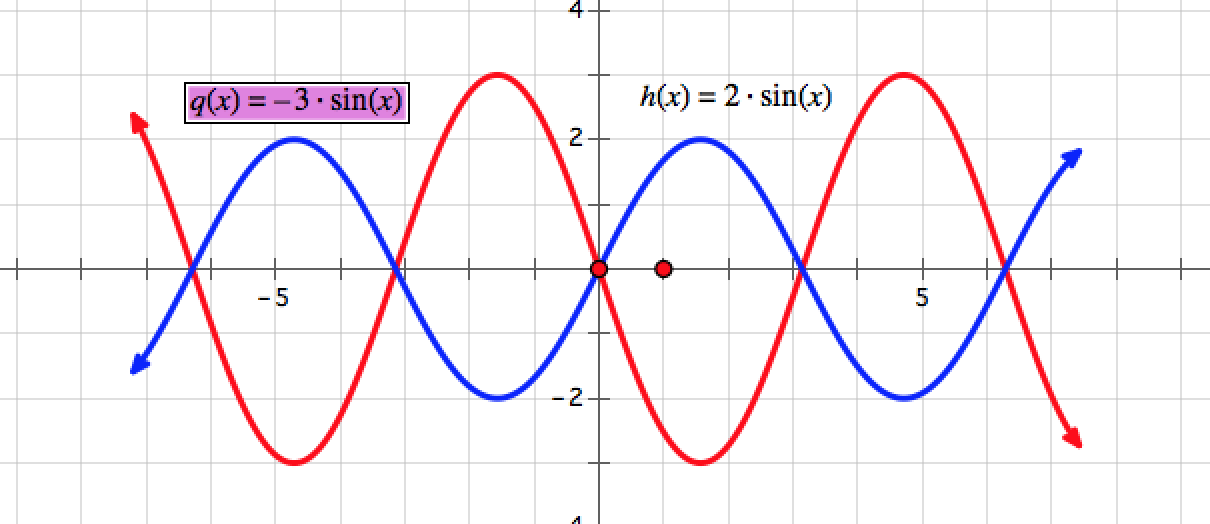
The multiplier *a,* which is outside the sine function tells us how high and how low the curve goes, and is called the amplitude. Actually, the **amplitude** is defined as “half the distance from a peak to a trough.” In our general equation:

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The reason it is the absolute value is because *a* could be negative, but we still refer to amplitude as a positive value.

The normal sine curve has *y* - values from *- 1 to + 1.* Thus, any sine curve will have *y* - values from *- a to + a.*  As an example, graphed below are

and



As can be seen, the multiplier *2* stretches the sine curve to go from *- 2 to + 2.* The multiplier *3* stretches the curve to run from *- 3 to + 3,* but also being negative, flips it over the *x -axis.* Even though it is flipped, its amplitude is still *+ 3.*

2. **Period**

We have defined the **period** as the time it takes for the curve to complete one cycle, or stated slightly differently, to get back to the same *y -* value AND begin the pattern over again.

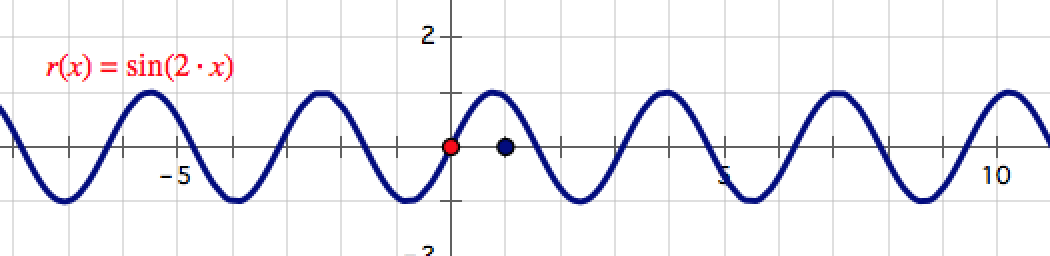
The period of the normal sine curve is *2π, which is one time around the unit circle.* We find the altered period by using the *b -* valuefrom our standard equation. The period of any curve we want to graph is *2π* divided by *b.*

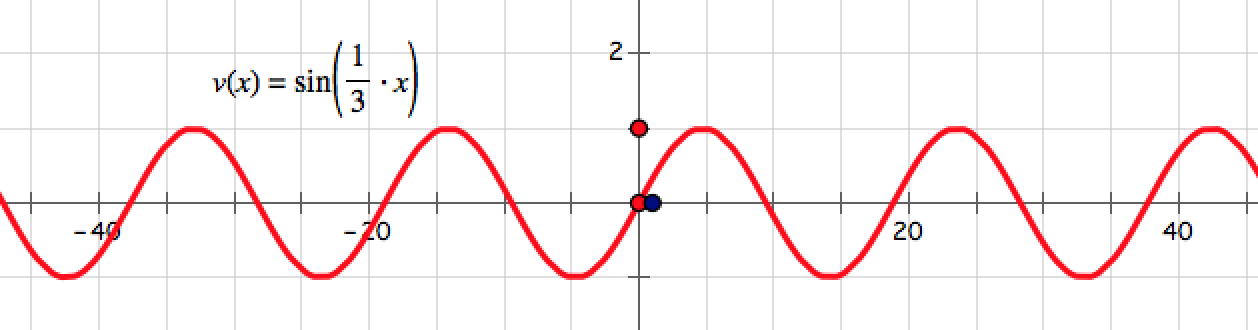
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Thus, if *b* is *2,* the period is *2π/b* = *2π/2* or *π.* If *b* is *1/3,* the period is:

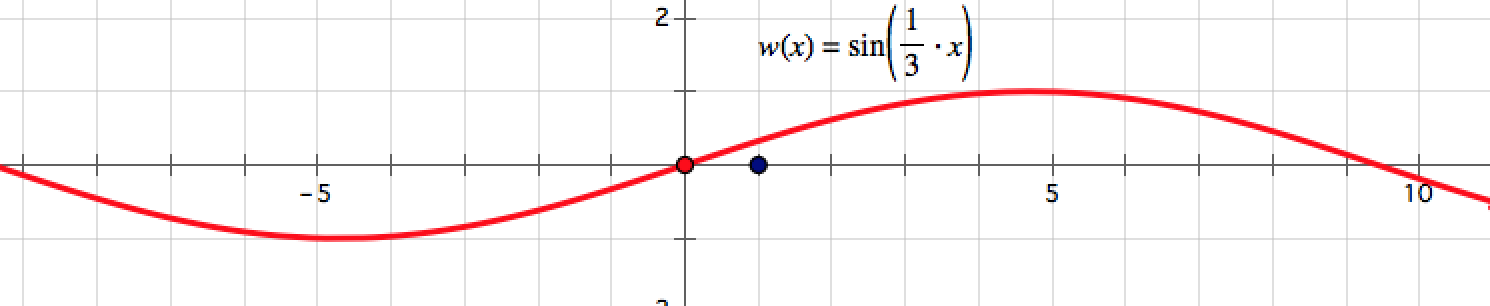
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Below, both curves, with *b =* 2 and *b = 1/3,* are graphed:

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Although these two curves look the same, if one looks closely at the scale on the *x-*axes*,* the scales are very different. The lower curve has a period six times the period of the upper curve.If graphed on the same scale as the upper curve, the lower curve looks like this:

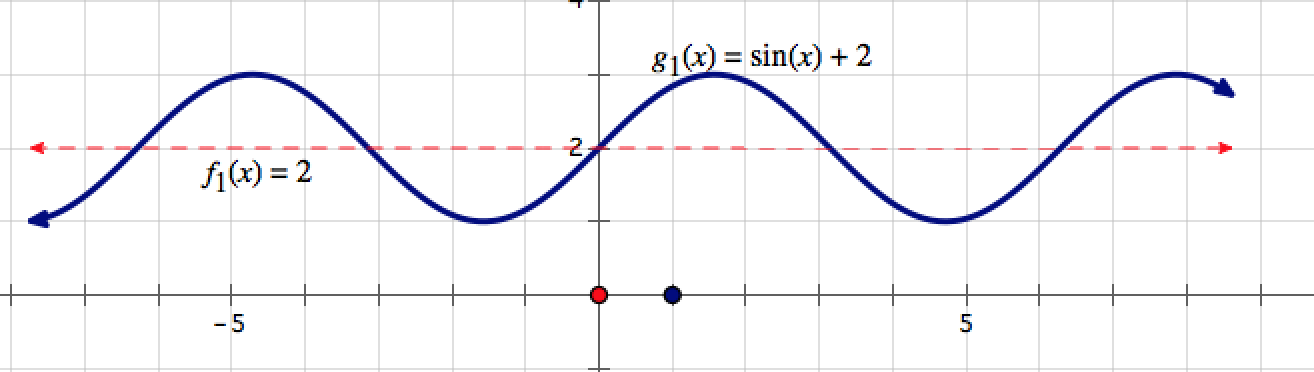


Once we have the period, graphing is easiest if we divide the period by *4* andlocate those values on the *x -*axis. At each quarter period, there will be a maximum, minimum or *x -* intercept.

3. **Centerline**

Thinking back to our general sine curve, , the value *k* moves the curve up or down. This is the same as when we studied translations of any function, *f(x)*. I refer to it as the “centerline,” because whatever the value of *k*, the entire curve is moved that distance, which also means, rather than oscillating around the *x -* axis*,* the curve will oscillate around the line *y = k.*

For example, if we have the curve of , which has our normal amplitude of *1,* our curve will go *1* above *y = 2,* and *1* below *y = 2.*

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This curve, , is exactly the same as *y = sinx,* same amplitude and same period. The only difference is that the whole thing is moved up by two, to oscillate around *y = 2.* I find it very helpful to dot in the centerline at *y = k,* to help locate the curve. It makes it easier to represent the amplitude and one can put the period values on the centerline.

One aspect that is valuable to note about the transformed curve is that the origin, which is the point where the normal sine curve begins, is just moved to *(0, 2).* Recognizing the relocation of this starting point helps us deal with the horizontal shift. Recall, the cosine curve starts at *(0, 1).*

4. **Horizontal Shift**

The last piece, and potentially the most difficult, is the horizontal shift. We know from our study of translations of functions that *f(x - h)* is the same as *f(x)* only shifted to the right by *h,* and *f(x + h)* is the same as *f(x)* shifted *h* to the left. The same holds for sine curves. What causes confusion is having the *b* value inside the function. Before trying to evaluate the horizontal shift, it is necessary to have the function in the form , NOT in the form . This is very important. Let’s look at an example:

The equation is in the correct form to recognize *b = 4,* and the horizontal shift is *π* to the right.

The equation does not tell us the horizontal shift. It is necessary first to factor out the *4:* ***.***Now we see that the *b* value is *4,* and the horizontal shift is *π/4.*

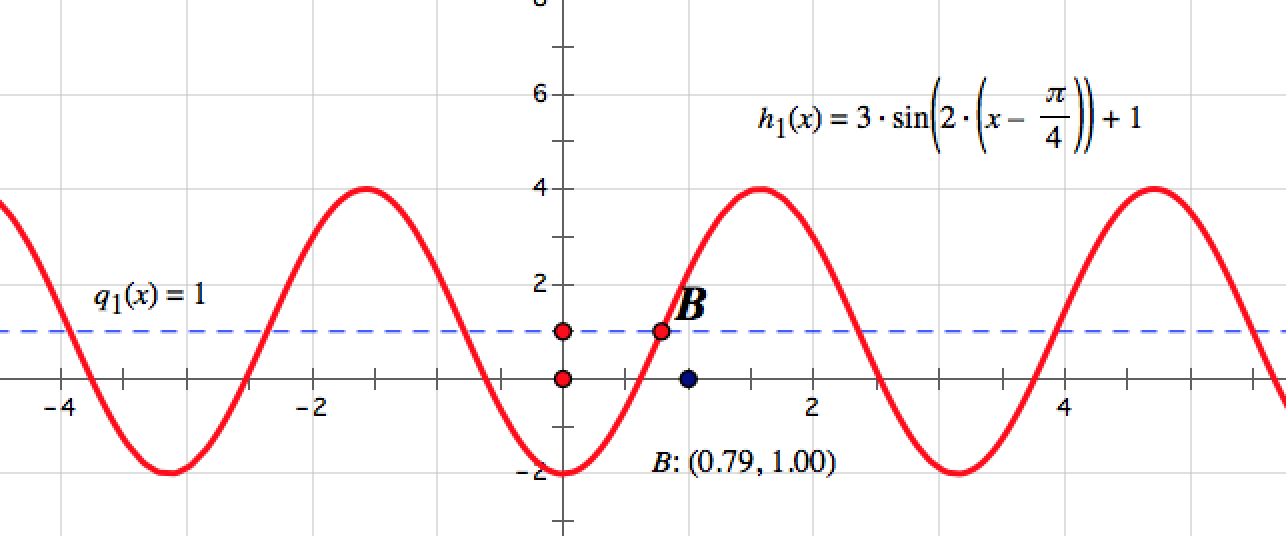
Let’s graph the following function:

1. The amplitude is *3.* The curve goes *3* above and *3* below the centerline.

2. The period is . This tells us that every there will be a maximum, a minimum or a centerline intercept.

3. The vertical shift is *+ 1,* so the centerline is at *y = 1.* If there were no horizontal shift, the curve would start at *(0, 1)*.

4. There is a horizontal shift of to the right. Thus, the starting point shifts to the right to the point . (Recall, ).

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Using these tools, we can comfortably graph any sine or cosine function.